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TURBULENT PRANDTL NUMBER IN THERMALLY STRATIFIED SHEAR FLOWS OF AIR

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Abstract—Turbulent Prandtl number and eddy viscosity distribution in the thermally stratified turbulent boundary layer of an air flow are found to be functions of distance from the wall and thermal stabilities of the flow field. Turbulent Prandtl number in thermally stable shear flow of air is generally greater than that in thermally unstable shear flow of air.

NOMENCLATURE

c_p, specific heat of air at constant pressure [cal/°Cg];
g, gravitational acceleration [cm/s^2];
H, heat flux in the vertical direction [cal/cm^{2}s];
k, von Kármán constant;
k_h, molecular thermal conductivity [cal/cms °C];
k_t, total thermal conductivity [cal/cms °C];
K_h, eddy thermal diffusivity [cm^2/s];
K_m, eddy viscosity [cm^2/s];
L', length scale, $T_m A_1^2/gA_2$ [cm];
N, total number of data points in a profile;
Pr_e, eddy Prandtl number;
Pr_l, laminar (molecular) Prandtl number;
Pr_t, turbulent (total) Prandtl number;
R, dimensionless lapse rate;
Ri, Richardson number;
RTT, dimensionless temperature, $(T_i - T_m)/T_* - \beta'(y_i - y_m)/L'$;
RUU, dimensionless velocity, $k(U_i - U_m)/u_* - \beta'(y_i - y_m)/L'$;
RZZ, dimensionless height,

$$\ln y_i - \frac{1}{N}\sum_{i=1}^{N} \ln y_i;$$

S, dimensionless wind shear;
t^+, dimensionless temperature, $-(T - T_w)\, c_p \rho u_*/H$;
T, local mean temperature [°K];
T_*, friction temperature, $-H/c_p\rho k u_*$ [°C];
T_w, wall temperature [°K];
u^+, dimensionless velocity U/u_*;
u_*, friction velocity [cm/s];
$\overline{uv}$, turbulent shearing stress [cm^2/s^2];
U, local mean velocity [cm/s];
$\overline{vt}$, covariance between vertical velocity and temperature [°C-cm/s];
y, distance from the wall [cm];
y^+, dimensionless distance from the wall, yu_*/ν;
β', arbitrary constant;
ϵ_h^+, dimensionless total thermal conductivity;
ϵ_u^+, dimensionless total viscosity;
μ, molecular dynamic viscosity [g/cms];
μ_t, total viscosity [g/cms];
ν, molecular kinematic viscosity [cm^2/s];
ρ, density of air [g/cm^3];
τ, total shear stress in the flow direction [g/cms^2];
ϕ, dimensionless eddy viscosity, K_m/ν;

$(\ )_i$, the variable at height y_i;
$(\ )_m$, mean value averaged over the profile.

1. INTRODUCTION

THE SOLUTION of the partial differential equation of heat transfer in a thermally stratified turbulent boundary layer requires certain assumptions and approximations. The first three assumptions are the mean velocity distribution, the mean temperature distribution and the variation of the total thermally conductivity.

Assuming logarithmic profiles for the mean velocity and the mean temperature distribution, Spalding [1] obtained a constant turbulent Prandtl number, $Pr_t = 0{\cdot}887$. He also assumed a power series and exponential representation of the total thermal conductivity in terms of the mean velocity. Patankar [2] suggested power law profiles for the mean velocity and the mean temperature distribution. Patankar and Spalding [3] then assumed two-component velocity and temperature profiles which are equivalent to the log-linear profiles. Baker [4] also assumed a power series in terms of the mean velocity for the total thermal conductivity and found the heat transfer from a smooth wall into a steady, uniform-property turbulent boundary. He considered a constant turbulent Prandtl number throughout the boundary layer. As shown by Kestin and Richardson [5], Johnson [6] and Swinbank [7], the turbulent Prandtl number in the boundary layer varies with position and experiments. They questioned [5] the wisdom of using a velocity profile obtained in thermally neutral shear flows to determine a turbulent Prandtl number and an eddy thermal conductivity in thermally stratified shear flows. The turbulent Prandtl number is not well known at the present time. There is a lack of agreement about its value and its variation in the thermally stratified turbulent boundary layer. Therefore, it must be determined experimentally in various thermal stratifications.

The total viscosity in thermally neutral turbulent boundary layer can be measured experimentally. However, when the turbulent shear flow is thermally stratified, it may not be measurable directly. Spalding [1] assumed that the eddy viscosity was a function of only the dimensionless velocity which, in turn, could be transformed to position. This function was given as a power series and exponential of the velocity. In experiments performed by one of the authors of this article [8], it was evident that the turbulent shearing stress in thermally stratified wall layer was not independent of the thermal stability of the stream. Consequently, the eddy viscosity should also depend on the thermal stability.

The existence of a similarity between the mean velocity and the mean temperature profiles in a thermally stratified turbulent shear flow was examined in [9–11]. The similarity is usually considered to hold only in the constant shear layer where the turbulent mixing predominates. In a neutral flow, a universal velocity profile or the so-called "law of the wall" [12] can prescribe the mean velocity distribution in this layer. However, the mean velocity distribution in a thermally stratified turbulent shear flow cannot be described by this universal profile for the same region. It is represented more accurately by a log-linear profile [13] such as

$$U(y) = A_1 \ln y + B_1 y + C_1, \qquad (1)$$

where A_1, B_1 and C_1 can be determined experimentally by means of the least squares fitting of the measured velocity distribution. These coefficients are in fact dependent upon the thermal stability of the flow field and B_1 approaches zero as the flow approaches a neutral stratification.

Similarly, the mean temperature distribution is given by

$$T(y) = A_2 \ln y + B_2 y + C_2, \qquad (2)$$

where A_2, B_2 and C_2 can also be determined by the least squares fitting of the measured temperature distribution and they also depend on the thermal stability of the flow field.

Consequently, the turbulent Prandtl number and the eddy viscosity distributions in the layer can be calculated if the mean velocity and the mean temperature distributions in the constant shear layer of thermally stratified shear flows are approximated by these log-linear profiles. The turbulent Prandtl number thus obtained varies substantially with the thermal stability in the constant shear layer of thermally stratified shear flows of air. The motivation for this study was to show this variation.

2. BASIC EQUATIONS

In this section the turbulent Prandtl number, eddy viscosity and eddy thermal diffusivity will be formulated.

2.1 *Derivation of the turbulent Prandtl number*

The logarithmic term in equations (1) and (2) can be eliminated to yield

$$T = \frac{A_2}{A_1}U - \left[\frac{A_2}{A_1}B_1 - B_2\right]y - \left[\frac{A_2}{A_1}C_1 - C_2\right].$$

Differentiating the above equation with respect to height y gives

$$\frac{dT}{dy} = \frac{A_2}{A_1}\frac{dU}{dy} - \left[\frac{A_2}{A_1}B_1 - B_2\right]. \qquad (3)$$

The gradient of temperature and velocity are related to the heat flux in the vertical direction and the shearing stress in the flow direction, respectively, as follows:

$$H = -k_h\frac{dT}{dy} + c_p\rho\overline{tv} = -(k_h + c_p\rho K_h)\frac{dT}{dy} = -k_t\frac{dT}{dy}$$

and

$$\tau = \mu\frac{dU}{dy} - \rho\overline{uv} = (\mu + \rho K_m)\frac{dU}{dy} = \mu_t\frac{dU}{dy}$$

By substituting the above equations and the derivative of equation (1) into equation (3) and by rearranging, it yields the turbulent Prandtl number as

$$Pr_t = \frac{c_p\mu_t}{k_t} = -\frac{c_p\tau}{H}\left[\frac{A_2}{A_1} + \frac{(A_1B_2 - A_2B_1)\,y}{A_1(A_1 + B_1y)}\right].$$

Assuming that $\tau = \rho u_*^2$ and $H = -c_p\rho u_* kT_*$, the above equation can be rewritten as

$$Pr_t = \left(\frac{u_*A_2}{kT_*A_1}\right)\left(\frac{1 + B_2y/A_2}{1 + B_1y/A_1}\right). \qquad (4)$$

For logarithmic profiles, B_1 and B_2 in equation (4) are equal to zero and consequently the turbulent Prandtl number for a neutral flow is given by

$$Pr_t = \frac{u_*A_2}{kT_*A_1}. \qquad (5)$$

By using logarithmic profiles for the mean velocity and the mean temperature distributions in a fully developed turbulent flow as

$$U = 2{\cdot}5\,u_*\ln y + C_1 \qquad (6)$$

and

$$T = 2{\cdot}22\,kT_*\ln y + C_2, \qquad (7)$$

Spalding [1] obtained a constant value for the turbulent Prandtl number, $Pr_t = 0{\cdot}887$. Substituting $A_1 = 2{\cdot}5\,u_*$ and $A_2 = 2{\cdot}22\,kT_*$ into equation (5) will yield the above value.

Assuming that

$$\frac{u_*A_2}{kT_*A_1} = 0{\cdot}887,$$

equation (4) is rewritten as

$$Pr_t = 0{\cdot}887\frac{1 + B_2y/A_2}{1 + B_1y/A_1}. \qquad (8)$$

Equations (1) and (2) can also be rewritten as

$$U(y) = \frac{u_*}{k}\left[\ln y + \frac{B_1}{A_1}y\right] + C_1 = \frac{u_*}{k}f_1(y) + C_1 \qquad (1a)$$

and

$$T(y) = T_*\left[\ln y + \frac{B_2}{A_2}y\right] + C_2$$
$$= T_* f_2(y) + C_2. \quad (2a)$$

If the velocity and the temperature profiles are exactly similar, then the functions $f_1(y)$ and $f_2(y)$ converge to a single function which is called a universal function [13]. The Monin–Obukhov [13] similarity theory (hypothesis) is not and will not always be exactly correct. Nevertheless, it has proved [9, 10, 14] to give approximately correct profiles. It should, however, be pointed out that a spectral similarity between u' and t' at the same height is quite essential to a similarity between the mean-quantity profiles, because the mean-quantity profiles are closely related to the turbulent transfer mechanism of momentum and temperature.

Equation (4) in conjunction with equations (1a) and (2a) leads to

$$Pr_t = \frac{1 + B_2 y/A_2}{1 + B_1 y/A_1}. \quad (9)$$

The difference between equations (8) and (9) is due to the fact that equation (5) assumes the magnitude of unity when the A's are substituted from equations (1a) and (2a), but it is equal to 0·887 when the A's are furnished by equations (6) and (7). According to Kestin and Richardson [5], the turbulent Prandtl number for a turbulent pipe flow is always smaller than unity. Nevertheless, based on the Lagrangian description of the eddy motion, Tien [15] proposed that $Pr_t = 1$. It is questionable up to this point what value one should assume for equation (5). However, it is obvious from equation (4) that Pr_t is not constant across the turbulent boundary layer.

Defining the dimensionless wind shear, S, and the dimensionless lapse rate, R, respectively, [16] as

$$S = \frac{ky}{u_*}\frac{dU}{dy},$$

and

$$R = \frac{y}{T_*}\frac{dT}{dy},$$

one has, for log-linear profiles in the forms of equations (1a) and (2a),

$$S = 1 + B_1 y/A_1$$

and

$$R = 1 + B_2 y/A_2.$$

Thus, equation (4) can be written as

$$Pr_t = \frac{u_* A_2}{k T_* A_1}\frac{R}{S}. \quad (10)$$

By definition, the ratio R/S is given by

$$\frac{R}{S} = \frac{u_*}{kT_*}\frac{dT/dy}{dU/dy} = \frac{-\overline{uv}\, dT/dy}{-\overline{tv}\, dU/dy} = \frac{K_m}{K_h} = Pr_e. \quad (11)$$

Therefore, equation (10) also states that the turbulent Prandtl number is approximately equal to the eddy Prandtl number. The first term of equation (4), which is the turbulent Prandtl number for a neutral flow, may be equal to unity. However, this does not necessarily mean that the turbulent Prandtl number in thermally stratified flow is unity as Kestin and Richardson [5] casted doubt that the presence of a thermal field would not affect the law of the wall [1].

2.2 *Derivation of eddy viscosity and eddy thermal diffusivity*

Defining the dimensionless total viscosity and the dimensionless total thermal conductivity, respectively, as

$$\epsilon_u^+ = \frac{\mu_t}{\mu} = 1 + \phi(u^+) \quad (12)$$

and

$$\epsilon_h^+ = \frac{k_t}{c_p\mu} = \frac{1}{Pr_l} + \frac{1}{Pr_e}\phi(u^+), \tag{13}$$

where

$$\phi(u^+) = K_m/\nu, \tag{14}$$

one obtains for the constant shear layer

$$\phi(u^+) = \frac{(A_1k)^2}{\nu\, dU/dy} - 1. \tag{14a}$$

The molecular kinematic viscosity ν at atmospheric pressure is a weak function of the air temperature and according to data given by Schlichting [17] it can be approximately expressed in an exponential function for air temperatures from $-10°C$ to $60°C$ as

$$\nu = 0{\cdot}1302 \exp[0{\cdot}00665T], \tag{15}$$

where T is in degree C. Assuming that von Kármán constant k in equation (14a) is equal to 0·4 the eddy viscosity is given by

$$K_m = \frac{0{\cdot}16\, A_1^2}{dU/dy} - 0{\cdot}1302 \exp[0{\cdot}00665T]. \tag{16}$$

The velocity gradient in the vertical direction can be approximated by the finite difference technique and A_1 is given by equation (1). Thus, the eddy viscosity in the constant shear layer of a thermally stratified shear flow can be determined once the mean velocity and the mean temperature profiles are measured. If the von Kármán constant k is not exactly equal to 0·4 [18] throughout the layer, then equation (16) will not give accurate results. The dimensionless eddy viscosity, as defined by equation (14), can also be found by means of equation (14a). Dimensionless eddy viscosity in the constant shear layer is largely dependent upon the thermal stability of the flow field as well as upon the distance from the boundary wall. The eddy viscosity distribution in the wall region of the neutral boundary layer flow, as shown in Fig. 7–41 of [19], is a power profile of a dimensionless distance from the wall where the power assumes a value of 3 or higher. It reveals that the eddy viscosity is much greater than the molecular kinematic viscosity in the fully developed turbulent region, $yu_*/\nu > 30$, at $Re_\delta = 8 \times 10^4$. Therefore, the second term on the right hand side of equation (16) is negligible in the constant shear layer of a neutral flow. The dimensionless eddy viscosity in the layer is also much greater than unity. Hence, the total viscosity in the layer is practically equal to the eddy viscosity such that

$$\epsilon_u^+ \approx \phi(u^+). \tag{12a}$$

The molecular (laminar) Prandtl number for air at atmospheric pressure and temperature in the range from 0° to 90°C is approximately constant and is equal to 0·72 [20]. Thus, the dimensionless total thermal conductivity can be determined from equation (13) as

$$\epsilon_h^+ \approx 1{\cdot}39 + \phi/Pr_t.$$

By definition the turbulent (or total) Prandtl number is also given by

$$Pr_t = \epsilon_u^+/\epsilon_h^+.$$

Therefore, the first term on the right hand side of equation (13) is negligible and the dimensionless total thermal conductivity in the layer is practically furnished by

$$\epsilon_h^+ \approx \phi/Pr_t. \tag{13a}$$

The eddy thermal diffusivity in this layer can be found from the relation that

$$Pr_t \approx Pr_e = K_m/K_h,$$

so that

$$K_h \approx K_m/Pr_t.$$

Assuming that the total Prandtl number in the viscous layer (laminar sublayer) or the buffer layer (transition zone) is constant† but that in the constant shear layer and the outer

† One may assume that the total Prandtl number is equal to the laminar Prandtl number in the viscous layer but it is an unknown function of y^+ in the other layers.

layer (wake-like region) it is a function of y or U and that all variables are independent of x, the dimensionless temperature distribution may then approximately be given by [1]

$$t^+ = \int_0^{14} \frac{\epsilon_u^+}{\epsilon_h^+} du^+ + \int_{14}^{u^+} \frac{\epsilon_u^+}{\epsilon_h^+} du^+$$

$$= 14\, Pr_t + \int_{30}^{y^+} Pr_t(y^+) \frac{df}{dy^+} dy^+, \quad (17)$$

where the point $u^+ = 14$ corresponding to $y^+ = 30$ is the upper boundary of the buffer layer and the unknown functions $Pr_t(y^+)$ and $u^+ = f(y^+)$ must be determined experimentally. Both functions depend, to some extent, on the thermal stability of the flow field. Therefore, the mean velocity distribution function in thermally stratified shear flows can be drastically different from that of the neutral flow as revealed in [8]. It may be suggested that the mean velocity distribution assumes a log-linear profile in the constant shear layer and it changes to a velocity defect profile in a form similar to equation (7-85a) of [19] in the outer region. The wake function w and the constant term may assume different values from those given depending on the thermal stability of the flow field.

3. RESULTS AND DISCUSSION

Mean velocity and temperature distributions in thermally stratified shear flows measured in a wind tunnel [9] and Project Prairie Grass [21] are presented in Fig. 1. The thermal and momentum boundary-layer thickness at the test section in the wind tunnel were both approximately equal to 70 cm, but here only the constant shear layer of approximately 8 cm in height is considered. The shearing stress and the friction temperature in this layer are not measured directly, but are determined by the least squares fitting of the measured mean velocity and the mean temperature profiles to equations (1a) and (2a), respectively.

Spalding [1] suggested that the turbulent Prandtl number might be constant and equal to

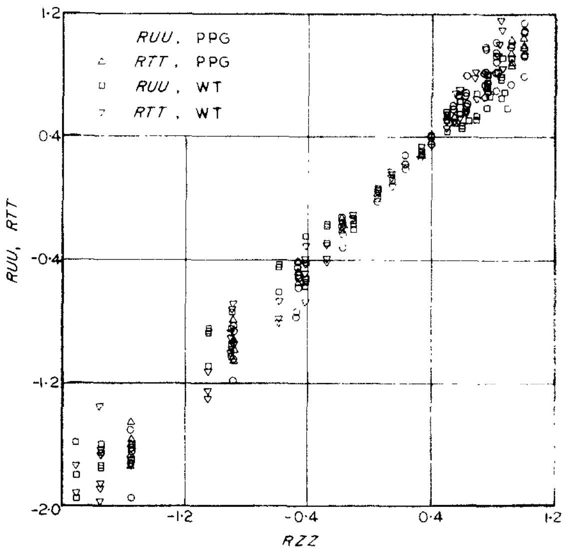

FIG. 1. Log-linear profiles of mean velocity and mean temperature for heights from 0·5 to 8·2 cm (Wind-Tunnel [9]) or from 25 to 750 cm (PPG, [21]).

0·887. However, equation (8) shows to the contrary that it varies with height. Figures 2–5, which are plots of equation (9), show the variation of turbulent Prandtl number in thermally stratified boundary layers (inner layers). Wind tunnel data are shown in Figs. 2 and 3, while field test data are shown in Figs. 4 and 5. Both the laboratory and the field test data show that the turbulent Prandtl number in thermally stable flows is generally greater than that in thermally unstable flows. This conclusion is in good agreement with the results of McVehil [22]. Johnson [6] also showed that the local turbulent Prandtl number was not constant across the boundary layer although he measured it at a section where the flow was not fully developed. While the data shows that the turbulent Prandtl number varies with height, it does not follow the fourth root of the dimensionless height suggested by Swinbank [7].

When inversion or stable stratification predominates, air pollution in an area can become serious. Thermally stable stratification in air flows is usually characterized by a large turbulent Prandtl number, $Pr_t \gg 1$, and it occurs near

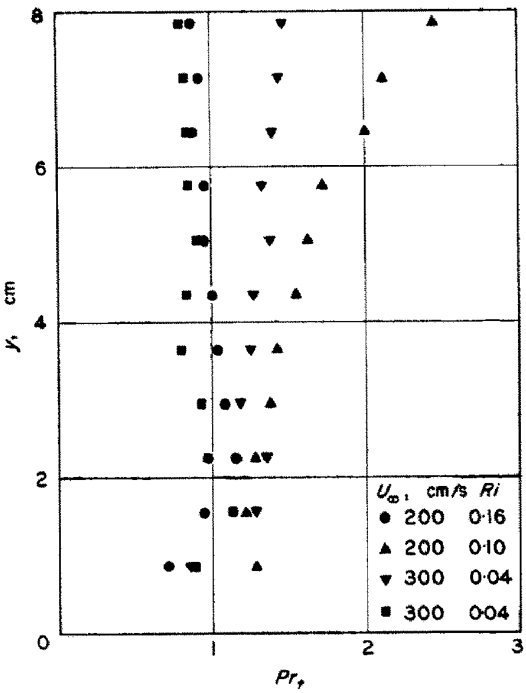

FIG. 2. Variation of turbulent Prandtl number across the constant shear layer of thermally stable flows in a wind tunnel.

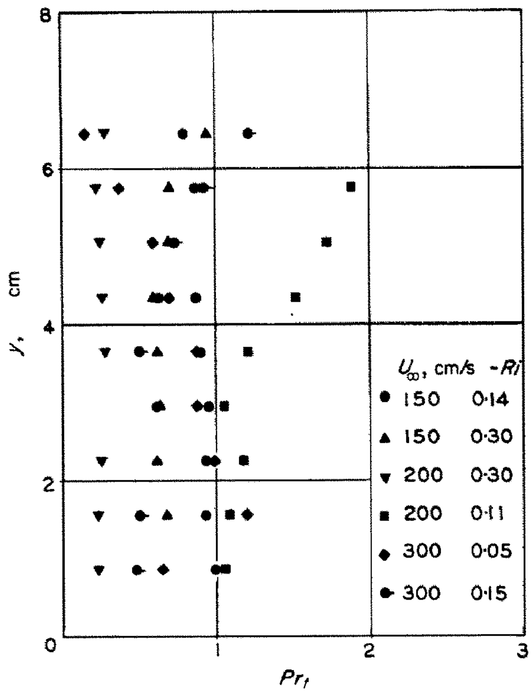

FIG. 3. Variation of turbulent Prandtl number across the constant shear layer of thermally unstable flows in a wind tunnel.

the ground at night with clear skies. Referring to equation (11), a large turbulent Prandtl number means that the dimensionless lapse rate is much greater than the dimensionless wind shear. In other words, the eddy thermal diffusivity is much smaller than the eddy viscosity. Therefore, the mechanism of turbulent transfer of momentum is much greater than that of turbulent transport of heat in the constant shear layer of thermally stable flow. For thermally unstable flow, the opposite phenomena occurs.

Distribution of the eddy viscosity, the dimensionless total viscosity, and the dimensionless total thermal conductivity in the constant shear layer of thermally stratified air flows are shown in Figs. 6 and 7. They are typical variations of these properties in a thermally stratified turbulent shear flow of air in the laboratory (Fig. 6) or in the field (Fig. 7). The dimensionless total viscosity in the constant shear layer is obviously much greater than unity and, therefore, is approximately equal to the dimensionless eddy viscosity as predicted by equation (12a). The dependence of the dimensionless total viscosity on the dimensionless velocity must be examined in the light of equation (20) of [1] or equation (14) of [23]. Typical variations of the dimensionless total viscosity versus the dimensionless velocity in the constant shear layer of thermally stratified shear flows are shown in Fig. 8. That the effect of the temperature distribution on the mean velocity profile in the layer is not negligible implies the functional dependence of the dimensionless total viscosity not only upon the dimensionless velocity but also upon the thermal stability of the flow field. If this is the case, then equation (12) is superseded by

$$\epsilon_u^+ = 1 + \phi(u^+, Ri)$$

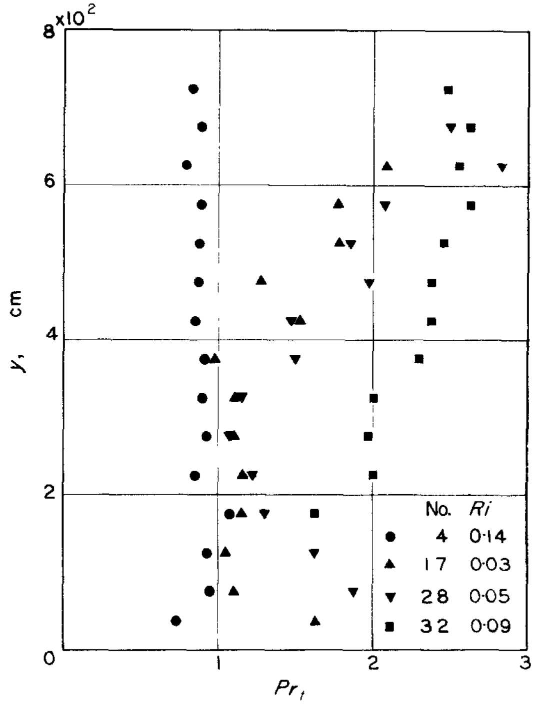

FIG. 4. Variation of turbulent Prandtl number across the constant shear layer of thermally stable flows in the field (PPG [21]).

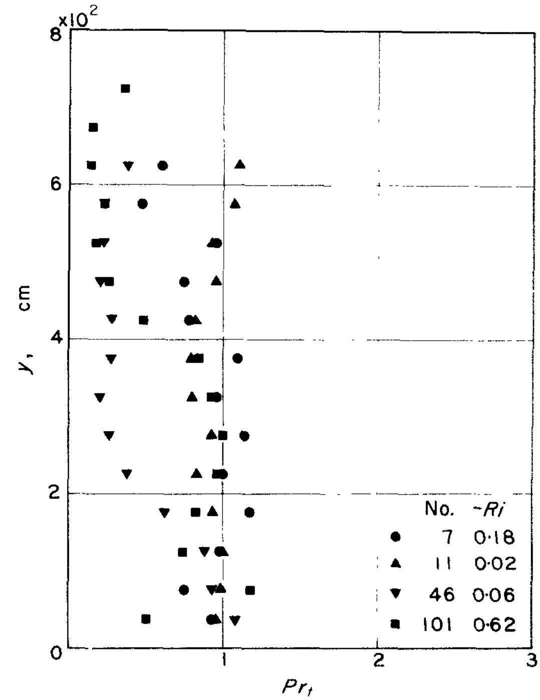

FIG. 5. Variation of turbulent Prandtl number across the constant shear layer of thermally unstable flows in the field (PPG [21]).

where

$$Ri = \frac{2g}{T_1 + T_2} \frac{\dfrac{(T_2 - T_1)}{(y_2 - y_1)}}{\left[\dfrac{(U_2 - U_1)}{(y_2 - y_1)}\right]^2}.$$

The subscripts 1 and 2 refer to the boundaries of the constant shear layer.

The dimensionless total thermal conductivity as defined by equation (13) is also apparently much greater than the reciprocal of the laminar Prandtl number of air in the layer. Thus, equation (13a) will give a moderately accurate value of it in the constant shear layer. The dependence of the total thermal conductivity on the dimensionless velocity should also be studied. The eddy thermal diffusivity in the layer can be obtained very easily by means of equation (11).

The mean velocity distribution in a neutral turbulent boundary layer is approximately given by equations (15) of [23] or equation (10) of [24]. It is doubtful that the mean velocity distribution in thermally stratified shear flows will assume the same functional dependence on y as the neutral flow does. Therefore, the functional form of $u^+ = f(y^+)$ for thermally stratified shear flows must be determined experimentally.

4. CONCLUSIONS

Turbulent Prandtl number in the constant shear layer is not necessarily always constant. It depends on the thermal stability of the flow field and may vary with height. The turbulent Prandtl number in thermally stable turbulent shear flow is generally greater than that in thermally unstable turbulent shear flow of air.

The eddy viscosity as well as the eddy thermal diffusivity in the constant shear layer of air may be determined.

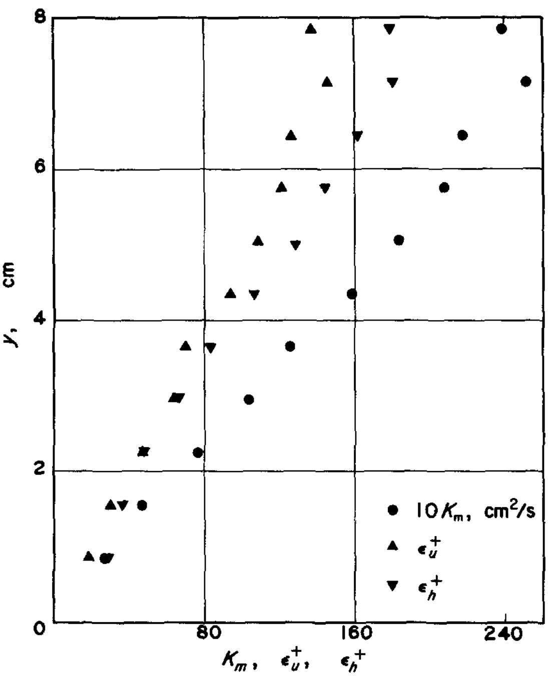

FIG. 6. Eddy viscosity, dimensionless total viscosity and dimensionless total thermal conductivity in a laboratory flow vs. height. The flow is thermally stable and the free stream velocity is 200 cm/s. $Ri = 0{\cdot}16$.

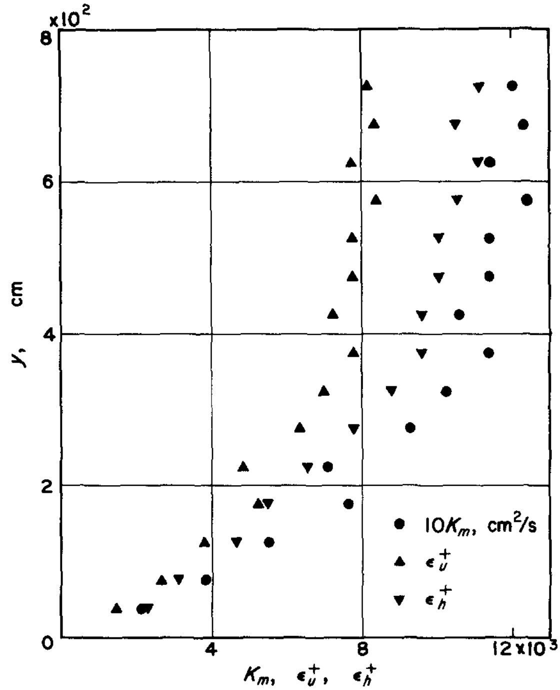

FIG. 7. Eddy viscosity, dimensionless total viscosity, and dimensionless total thermal conductivity in a stable field flow (No. 4, PPG [21]) vs. height. $Ri = 0{\cdot}14$.

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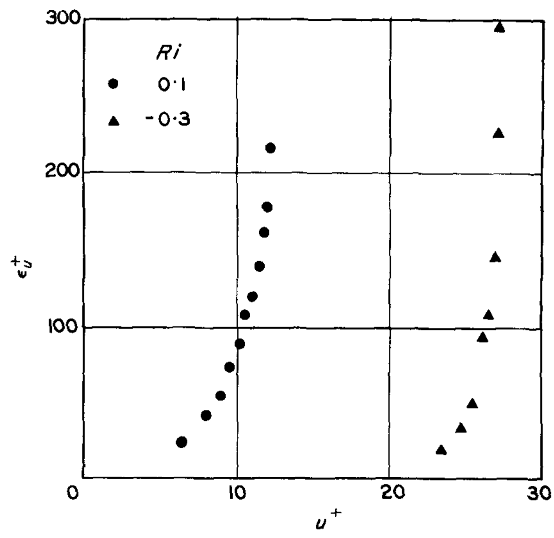

FIG. 8. Dimensionless total viscosity vs. dimensionless velocity for thermally stratified shear flows in a wind tunnel with a free stream velocity 200 cm/s.

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NOMBRE DE PRANDTL TURBULENT DANS DES ÉCOULEMENTS DE CISAILLEMENT D'AIR STRATIFIÉ THERMIQUEMENT

Résumé—On trouve que le nombre de Prandtl turbulent et la distribution de la viscosité turbulente dans la couche limite turbulente stratifiée thermiquement d'un écoulement d'air sont des fonctions de la distance à la paroi et des stabilités thermiques du champ d'écoulement. Le nombre de Prandtl turbulent dans un écoulement de cisaillement thermiquement stable d'air est généralement plus grand que celui dans un écoulement de cisaillement thermiquement stable d'air.

TURBULENTE PRANDTL-ZAHL IN THERMISCH AUSGERICHTETEN SCHERSTRÖMUNGEN BEI LUFT

Zusammenfassung—Es zeigte sich, dass die Verteilung der turbulenten Prandtl-Zahl und des Wirbelaustauches in der thermisch ausgerichteten turbulenten Grenzschicht einer Luftströmung Funktionen der Entfernung von der Wand und der thermischen Stabilitäten des Strömungsfeldes sind. Die turbulente Prandtl-Zahl in einer thermisch stabilen Scherströmung bei Luft ist gewöhnlich grösser als jene in einer thermisch instabilen Scherströmung bei Luft.

Аннотация—Найдено, что турбулентные числа Прандтля и распределение вихревой вязкости в термически расслоенном турбулентном пограничном слое воздушного потока являются функциями расстояния от стенки и тепловой устойчивости потока. Турбулентное число Прандтля в термически устойчивом потоке воздуха со сдвигом обычно больше *Pr* в термически неустойчивом потоке воздуха со сдвигом.